3. KOLOKVIJ IZ MATEMATIKE 1 2. DIO

OGLEDNI PRIMJERAK, RJEŠENJA

1. (i) $\lim\limits_{x\to 3}\dfrac{x^2-7x+12}{x^2-4x+3}=\dfrac{0}{0}=\lim\limits_{x\to 3}\dfrac{\cancel{(x-3)}(x-4)}{\cancel{(x-3)}(x-1)}=\dfrac{3-4}{3-1}=\dfrac{-1}{2}=-\dfrac{1}{2}$

$(x^2-7x+12):(x-3)=x-4$

$\underline{-x^2+3x}$

$-4x+12$

$\underline{4x-12}$

0

$(x^2-4x+3):(x-3)=x-1$

$\underline{-x^2+3x}$

$-x+3$

$x-3$

(ii) $\lim\limits_{x\to 0}\dfrac{(2-2\cos x)\sin x}{3x^3}=\lim\limits_{x\to 0}\dfrac{2\cdot 2\sin^2\frac{x}{2}\sin x}{3x^3}=4\lim\limits_{x\to 0}\dfrac{\frac{\sin^2\frac{x}{2}}{x^2}\cdot\frac{\sin x}{x}}{3}=$

$=\dfrac{4}{3}\lim\limits_{x\to 0}\dfrac{\sin^2\frac{x}{2}}{4\left(\frac{x}{2}\right)^2}\cdot\dfrac{\sin x}{x}=\dfrac{1}{3}$

(iii) $f(x)=\sqrt{\ln x+1}+\ln(\sqrt{x}+1)$

$f'(x)=\dfrac{1}{2\sqrt{\ln x+1}}\cdot\dfrac{1}{x}+\dfrac{1}{\sqrt{x}+1}\cdot\dfrac{1}{2\sqrt{x}}$

(iv) $f(x)=\arccos e^x+\sin\left(2x+\frac{\pi}{6}\right)$

$f'(x)=-\dfrac{1}{\sqrt{1-e^{2x}}}\cdot e^x+\cos\left(2x+\frac{\pi}{6}\right)\cdot 2$, $f'(0)=-1+2\cdot\cos\frac{\pi}{6}=$

$=-1+2\sqrt{3}$

2. - približno e^{x-x^2} za $x=1.03$

$f(x_0+h)\approx f(x_0)+f'(x_0)h$; $x_0=1$ $h=0.03$

(i) lineano,

$f(x_0)=e^{1-1}=e^0=1$

$f'(x)=e^{x-x^2}(1-2x)$, $f'(1)=e^0(1-2\cdot 1)=-1$

$f(1.03)\approx 1-1\cdot 0.03=1-0.03=0.97$

(ii) kvadratno $f(x_0+h)\approx f(x_0)+f'(x_0)h+\dfrac{f''(x_0)}{2}h^2$

$f''(x)=e^{x-x^2}(1-2x)^2-2e^{x-x^2}$

$f''(1)=(-1)^2-2\cdot 1=1-2=-1$

$f(1.03)\approx 1-1\cdot 0.03-\frac{1}{2}\cdot(0.03)^2=0.97-\frac{1}{2}\,0.0009$

- radijus povećan za 0.01, promjena opsega (približna)

(i) lineano: $O(r)=2\pi r$, $O(r_0+\Delta r)\approx O(r_0)+O'(r_0)\Delta r$

$\Rightarrow O(r_0+\Delta r)-O(r_0)\approx O'(r_0)\Delta r$

$O'(r)=2\pi \Rightarrow O'(r_0)=2\pi$

pa $O(r_0+\Delta r)-O(r_0)\approx 2\pi\Delta r=2\pi\cdot 0.01$

(ii) kvadratno:

$$O(r_0+\Delta r)-O(r_0)\approx O(r_0)+O'(r_0)\Delta r+\frac{O''(r_0)}{2}(\Delta r)^2$$

no $O''(r)=(2\pi)'=0$ pa je to isto kao za linearnu

3. (i) $x_0=0$ $\quad f(x)=\frac{2}{4-x}=\frac{2}{4}\frac{1}{1-\frac{x}{4}}$ pa imamo

$$f(x)=\frac{1}{2}\left(1+\frac{x}{4}+\left(\frac{x}{4}\right)^2+\dots+\left(\frac{x}{4}\right)^n+\dots\right)=$$

$$=\frac{1}{2}+\frac{x}{8}+\frac{x^2}{32}+\dots+\frac{x^n}{2\cdot 4^n}+\dots=$$

$$=\frac{1}{2}+\frac{x}{2^3}+\frac{x^2}{2^5}+\dots+\frac{x^n}{2^{2n+1}}+\dots$$

(ii) područje konv $\left|\frac{x}{4}\right|<1 \Rightarrow -4<x<4$

(iii) uz x^{100} stoji $\frac{f^{(100)}(0)}{100!}$ a u ovom redu

$$\frac{1}{2^{2\cdot 100+1}} \Rightarrow \frac{f^{(100)}(0)}{100!}=\frac{1}{2^{201}} \Rightarrow f^{(100)}(0)=\frac{100!}{2^{201}}$$

(iv)

$$f\left(\frac{1}{3}\right)=\frac{1}{2}+\frac{1}{3\cdot 2^3}+\frac{1}{3^2\cdot 2^5}+\dots+\frac{1}{3^n\cdot 2^{2n+1}}+\dots=\frac{1}{1-\frac{1}{12}}=\frac{1}{1}$$

4. zadana je $f(x)=\frac{4x}{x^2+4}$

(i) domena: $\mathbb{R}$, ne može biti x^2+4

(ii) nultočka: $x=0$

(iii) $f'(x)=\frac{4(x^2+4)-4x\,2x}{(x^2+4)^2}=4\frac{x^2+4-2x^2}{x^2+4}=4\frac{4-x^2}{4+x^2}$

$f'>0$ za $4-x^2>0 \Rightarrow x\in(-2,2)$

$f'<0$ za $4-x^2<0 \Rightarrow x\in(-\infty,-2)\cup(2,+\infty)$

(iv) $f'=0$ za $x=-2$, $x=2$

$$f''(x)=4\frac{-2x(4+x^2)-(4-x^2)\cdot 2x}{(x^2+4)^2}=8\frac{-4x-\cancel{x^3}-4x+\cancel{x^3}}{(x^2+4)^2}=$$

$$=\frac{-32x}{(x^2+4)^2}$$

$f''(2)=\frac{-64}{(4+4)^2}<0$ lok max $\quad f(2)=\frac{8}{4+4}=1$

$$f''(-2) = \frac{64}{(4+4)^2} > 0 \quad \text{lok min} \quad f(-2) = \frac{-8}{8} = -1$$

(v) $f'' > 0$ za $x < 0$ konveksna

$f'' < 0$ za $x > 0$ konk

$f'' = 0$ za $x = 0$ infl.

(vi) HA: $\lim\limits_{x \to \pm\infty} \frac{4x}{x^2+4} = 0\pm$

KA: nema

VA: nema